Week 11, disc 104

Wednesday, April 7, 2021 10:39 AM

5.3 Problems

Use Taylor's method of order two to approximate the solutions for each of the following two initial value problems.

Problem 1. $y' = te^{3t}$, $0 \le t \le 1$, $y(0) = 0$, $h = .5$

Problem 2. $y' = \sin(t) + e^{-t}$, $0 \le t \le 1$, $y(0) = 0$, $h = .5$

$$y_{i+1} = y_i + h f(t_i, y_i) + \underbrace{\frac{h^2}{2} f_t(t_i, y_i)}$$

Taylor method of order n

$$w_0 = \alpha, \quad \leftarrow y(0)$$

$$w_{i+1} = w_i + hT^{(n)}(t_i, w_i), \quad \text{for each } i = 0, 1, \ldots, N-1, \quad (5.17)$$

where

$$T^{(n)}(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i) + \cdots + \frac{h^{n-1}}{n!} f^{(n-1)}(t_i, w_i).$$

Euler's method is Taylor's method of order one.

$w_i \approx y(t_i)$

1) $\alpha = w_0 = 0$ $\quad t_0 = 0$

$h = .5$ $\quad t_i = t_{i-1} + h$

$f(t,y) = te^{3t}$

$f_t(t,y) := \partial_t(te^{3t}) = e^{3t} + t \cdot 3e^{3t} = e^{3t}(1+3t)$

$w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2} f_t(t_i, w_i)$

$w_0 = 0$

$w_1 = 0 + .5 \cdot 0 + \frac{.5^2}{2}(1)e^{3 \cdot 0} = \frac{1}{8}$

$w_2 \approx 2.646$

2) $f_t(t,y) = \cos(t) - e^{-t}$

$w_0 = 0$

$w_1 = .5$

$w_2 \approx 1.077$

Note:

$y = -\cos(t) - e^{-t} + C_0$

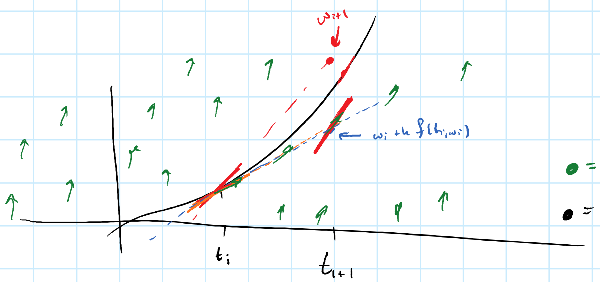
$0 = -1 - 1 + C_0$

$C_0 = 2$

$y(t) = -\cos(t) - e^{-t} + 2$

5.4 Problems

Problem 3. *Use modified Euler method to approximate the solution to the following initial-value problem and compare the results to the actual values:* $y' = 1 + y/t$, $1 \le t \le 2$, $y(1) = 2$, $h = .25$, $y(t) = t\ln t + 2t$.



$\bullet = f(t,x)$

$\bullet = y(t)$

Modified Euler Method

$$w_0 = \alpha, \quad \leftarrow y(0)$$

$$w_{i+1} = w_i + \frac{h}{2}[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))], \quad \text{for } i = 0, 1, \ldots, N-1.$$

$\text{int}_1 := f(t_i, w_i)$

$w_{i+1} = w_i + \frac{h}{2}(\text{int}_1 + \text{int}_2)$ $\qquad \text{int}_2 := f[t_{i+1}, w_i + h(f(t_i, w_i))]$

h	i	t	w	int1	int2			actual		absolute error
0.25	0	1	2	3	3.2			2		0
0.25	1	1.25	2.775	3.22	3.386667			2.778929		0.003929
0.25	2	1.5	3.600833	3.400555556	3.543413			3.608198		0.007364
0.25	3	1.75	4.468829	3.55361678	3.678617			4.479328		0.010498
0.25	4	2	5.372859	3.68642928	3.79754			5.386294		0.013436

Problem 4. *Repeat the above problem using Midpoint method.*

Problem 5. *Repeat the problem with Heun's method*

Problem 6. *Repeat the problem with Runge-Kutta of order four.*

$$w_0 = \alpha,$$

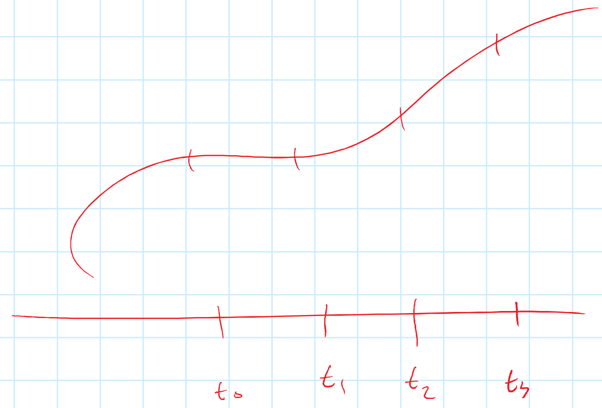
$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)\right), \quad \text{for } i = 0, 1, \ldots, N-1.$$

$$w_0 = \alpha$$

$$w_{i+1} = w_i + \tfrac{h}{4}\left(f(t_i, w_i) + 3\left(f\left(t_i + \tfrac{2h}{3}, w_i + \tfrac{2h}{3} f\left(t_i + \tfrac{h}{3}, w_i + \tfrac{h}{3} f(t_i, w_i)\right)\right)\right)\right),$$

for $\quad i = 0, 1, \ldots, N-1.$

$$w_0 = \alpha,$$
$$k_1 = hf(t_i, w_i),$$
$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right),$$
$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right),$$
$$k_4 = hf(t_{i+1}, w_i + k_3),$$
$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$



5.6 Problems

Problem 7. *Use two step Adams-Bashforth methods to approximate the solutions to the following initial-value problem. Use exact starting values and compare the results to actual values.*

$y' = te^{3t} - 2y$, $0 \le t \le 1$, $y(0) = 0$, $h = .2$. *Actual solution:* $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$

$$w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2,$$
$$w_{i+1} = w_i + \frac{h}{24}[9f(t_{i+1}, \underline{w_{i+1}}) + 19f(t_i, \underline{w_i}) - 5f(t_{i-1}, \underline{w_{i-1}}) + f(t_{i-2}, \underline{w_{i-2}})],$$

unknown

unknown.

$w_{-1} = y(t_{-1})$

$w_0 = y(0)$

$w_1 = \underbrace{\qquad\qquad}_{w_0,\ w_{-1}}$

$$w_0 = \alpha, \quad w_1 = \alpha_1,$$
$$w_{i+1} = w_i + \frac{h}{2}[3f(t_i, w_i) - f(t_{i-1}, w_{i-1})],$$